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Application of Modified Shapley Value in Gains Allocation of Closed-loop Supply Chain under Third-Party Reclaim

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Abstract

The closed-closed-loop supply chain is a complex system with many uncertain factors, but it is certain that the gains of the participants in the system after the formation of a coalition are greater than the gains without cooperation. In the process of applying the cooperative game model to get an income allocation scheme which maintains a stable coalition, the researchers of this paper found that classical solutions to cooperative games - Shapley values suffer unreasonable high uncertainty. Therefore, this paper modifies the Shapley value method, and then uses the modified method to solve the allocation problem of the closed-loop supply chain under the third-party reclaim mode. The results show that the modified Shapley value has lower uncertainty.

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Keyword: Closed-closed-loop supply chain; cooperative game; gains allocation; Modified Shapley value

1. Introduction

Closed-loop supply chain (CLSC) is such a supply chain that, in addition to the traditional supply chain activities, it also includes the reclaim, recycle and reuse of the used products. There are various types of closed-loop supply chains. Among them, the third-party reclaim mode is that the manufacturers or the distributors pay the third-party reverse logistics service providers to return the products [1]. Under the Third-party reclaim mode loop supply chain system, the participants include the manufacturers, the distributors and the recyclers, who are independent decision makers. Manufacturers provide the distributors with products for selling to the customers, and the recyclers reclaim the waste products from

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the consumers. The waste products are available for manufacturers to remanufacture [2]. Under this mode of closed-loop supply chain, the cooperation relationship among the participants is obvious. It is proved that the cooperation can reduce overall costs and increase revenue, so that cooperation is their common optimal choice. Reasonable gains allocation among the participants is the basis to maintain this cooperation. Therefore, it is a problem to be solved under Multiplayer cooperative game [3].

Multiplayer cooperative game is also known as the coalitional game. To maintain a stable coalition, a major premise is that each party of the coalition gets greater gains than they can get under non-cooperation. The purpose of solving the cooperative game is to seek such a stable gains allocation strategy that maintains a coalition. Relative to the non-cooperative game theory, cooperative game theory is not mature, and the three most fundamental problems are not well solved yet, namely, the solution of cooperative game, cooperative game solution structural stability and the formation of cooperative game solution mechanism. So far, Shapley value is considered a superiority solution to dynamic coalition [4]. Shapley value, named in honor of Lloyd Shapley, who introduced it in 1953, describes one approach to the fair allocation of gains obtained by cooperation among several actors [5]. In this paper, we modified the Shapley value method for its high uncertainty, and use a modified Shapley value method to solve the gains allocation problem under the third-party reclaim mode of the closed-loop supply chain.

2. Uncertainty of Shapley value in coalition game and its improvement

The definition of Shapley value implies that the participants outside coalition S form another coalition $N-S$ against S. Under this assumption, the coalition S's gain is certain. As Shapley participants are assumed to join a particular coalition randomly and are subject to uniform distribution, which is inconsistent with the reality. That is why the distribution of gains applying Shapley value needs further consideration [6].

In reality, with the formation of coalition S, the remaining $|N|-|S|$ participants may have different coalition states, causing different gains, and the result of the different coalition states of the $|N|-|S|$ will affect the gains of coalition S. That means the gains of the coalition S is not fixed, but is change with the coalition states of the remaining participants. The assumption of the Shapley value solution is only an ideal state. This paper argues that such an unreasonable assumption is the reason why Shapley Value is with higher uncertainty.

According to definition of Shapley value described with the language of probability theory, Shapley value is the mathematical expectation of each participant's contribution to the coalition S. Maintaining the basic premise of the concept, according to the above reasons, the uncertainty of the gain of coalition S causes the uncertainty of participant i's contribution to the coalition S, resulting in uncertain contributions of participants to coalition S. Therefore, need to reconsider the definition of participant i's contribution to the coalition S.

Define U_i^{N-S} as the coalition state set of participants belonging to $N-S$, where $i \in S$, and u_j^{N-S} is an element of U_i^{N-S}. Further, define $V(S|u_j^{N-S})$ as the gain of coalition S under the state of u_j^{N-S} and $p(S|u_j^{N-S})$ as the probability of coalition S coming into form under the state of u_j^{N-S}. Then we have $\sum_{u_j^{N-S} \in U_i^{N-S}} p(S|u_j^{N-S}) = 1$, and $p(S|u_j^{N-S})$ is positive correlated with the gains of the participants belong to $|N|-|S|$. Considering the participants are rational, we can define

$$p(S|u_j^{N-S}) = V(u_j^{N-S}) \div \sum_{u_j^{N-S} \in U_i^{N-S}} V(u_j^{N-S}) \tag{1}$$

Define $\Delta_i^{N-S}(S|u_j^{N-S}) = V(S|u_j^{N-S}) - V(S-\{i\}|u_j^{N-S})$ as the contribution of i to the coalition S under the state of u_j^{N-S}. According to total probability formula, get participant i's contribution to the coalition.

$$\Delta_i^{'}(S)=\sum_{u_j^{N-S}\in U_i^{N-s}} p(S|u_j^{N-S})\Delta_i^{N-S}(S|u_j^{N-S})=E(S|U_i^{N-S}) \quad (2)$$

The new Shapley value is the participant's contribution to the coalition, as

$$\varphi_i^{'}=\sum_{i\in S}P(S)\Delta_i^{'}(S)=\sum_{i\in S}P(S)E(S|U_i^{N-S}), i=1,2,...,n \quad (3)$$

It is obvious that the modified Shapley value satisfies Shapley's three prerequisites which proves a fair and reasonable allocation, and also meets Shapley's three axioms. That means the gains allocation based on the modified shapely value is also a reasonable solution to a cooperative game and it is expected that the modified one has an advantage of lower uncertainty. To demonstrate that, need to introduce an important property of conditional mathematical expectation:

Assume ζ and η are mutually dependent random variables, for any x, when $\zeta=x$, $E(\eta|\zeta=x)$ is the estimation of η with the minimum variance.

If the $E(\eta|\zeta=x)$ is denoted by the characteristic function of the random variable ζ. When $\zeta=x$, its value is $E(\eta|\zeta=x)$. In the definition, $E(\eta|\zeta=x)$ is a random variable, and its mathematic expectation is $E\eta=E(E(\eta|\zeta=x))$. And this relationship also applies in the discrete occasions.

Accordingly, (2) shows that the modified Shapley value is the value of contribution of participant i to the coalition S with minimum variance, because

$$(\Delta_i^{'}(S)-\varphi_i^{'})^2=(E(S|U_i^{N-s})-\varphi_i^{'})^2<(\Delta_i(S)-\varphi_i)^2 \quad (4)$$

According to the mathematical description of the uncertainty of Shapley value and（4）, get

$$R^{'2}=\sum_{i\in S}P(S)(\Delta_i^{'}(S)-\varphi_i^{'})^2\le R^2=\sum_{i\in S}P(S)(\Delta_i(S)-\varphi_i)^2 \quad (5)$$

Formula（5） shows that the modified Shapley value has lower uncertainty than the Shapley value.

3. Gains Allocation Strategy in Closed-Loop Supply Chain Based on Modified Shapley Value

Gains allocation is a critical issue in the closed-loop supply chain after the formation of cooperation coalition. The following is based on the above modified Shapley value to establish a gains allocation model of closed loop supply chain, and then gives a numeric example.

From the current point of view of research on closed-loop supply chain, the closed-loop supply chain is a complex system. However, the cooperation among the manufacturers, distributors and third-party recyclers has consistency of interests, thus creating a cooperative game is a rational choice. The modified Shapley value can be applied to establish gains allocation model. In order to facilitate further research and to simplify model parameters, the assumptions to the model are as follows:

(1) In the market, there is one manufacturer, one distributor, and one recycler, and they are all rational actors.

(2) Suppose the demand of the product market is: $q=tp^{-\theta}$, where t,θ are constants and $t>0$， $\theta>1$.

(3) Assume the reclaim rate of the product is 1.

(4) If the manufacturer doesn't cooperate with the recycler, the manufacturer's unit cost of product is c_0; If they co-operate, the manufacturer's manufacturing cost is related to the condition of the reclaimed products by the recycler. The unit cost of reclaiming by recycler is c_2. The reclaiming cost should be related to the condition of products, and also be related to the cost of manufacturing the product, when the product is new and when the product's manufacturing cost is high, the reclaiming price tends to correspondingly increase. The unit cost for distributor distributing products is c_1.

(5) Let ε denote the condition and also utilization rate of the reclaimed products, where $0<\varepsilon<1$. The more ε close to 1 the newer the reclaimed products; the more ε close to 0, the older the products. Due to the cost of reclaiming products are positively related to the condition of the reclaimed products and also positively related to the manufacturing costs, may assume the reclaiming cost $c_2=\varepsilon^2 c_0$. If without cooperation, the recycler sells the reclaimed products to the manufacturer with unit price εc_0. If the three participants form a coalition, the coalition's manufacturing cost is $c_0^{'}=(1-\varepsilon)c_0+c_0\varepsilon^2$.

In the closed-loop supply chain, the manufacturer is at the core position, and the distributor and the recycler are only related with the sales price of new products and the reclaiming price of used products, so their gains mainly depend on the manufacturer's decision. In the cooperation game, the manufacturer is the large participation and is also where the coalition risk is located.

To allocate the gains, it first needs to solve the characteristic function of each sate. Let N be the set of the participants of the closed-loop supply chain. *N*= {manufacturer, distributor, recycler} = {1, 2, 3}. Assume $V(\varnothing)=0$. The following considers the non-empty set of coalition.

It is quite outright to prove that the formation of coalition can improve their gains. It is omitted due to limited space. Then an optimal gains allocation strategy is needed to ensure the stability of the coalition. The above modified Shapley value is applied to calculate the allocation strategy. The steps are as follows: first of all, based on the modified Shapley value, calculate the conditional probability, conditional gains, and their characteristic functions of various states. Next, calculate the contribution of the participants to the coalition. Finally, calculate each participant's gain allocation according to its contribution to the coalition. Let the gains allocation of the manufacturer, the distributor and the recycler are φ_1, φ_2 and φ_3, respectively. The results are

$$\begin{aligned}\varphi_1 =& \frac{1}{3}\Bigg[\frac{2t}{\theta-1}(\frac{\theta}{\theta-1})^{-2\theta}(c_1+c_0)^{1-\theta} + \frac{t\times(c_1+c_0)^{1-\theta}}{\theta-1}(\frac{\theta}{\theta-1})^{-\theta}(1-(c_1+c_0)\times(\frac{\theta}{\theta-1})^{2-\theta}) \\ &+t(\frac{\theta}{\theta-1})^{2-2\theta}\left(\frac{\left(c_1+(1-\varepsilon-\varepsilon^2)c_0\right)^{2-\theta}}{\theta-1} - c_0(\varepsilon-\varepsilon^2)(c_1+c_0)^{1-\theta}\right) + 2t\left(\frac{\theta}{\theta-1}\right)^{-\theta}\left(\frac{\left(c_1+(1-\varepsilon+\varepsilon^2)c_0\right)^{1-\theta}}{\theta-1} - \frac{(c_1+c_0)^{1-\theta}}{\theta}\right)\end{aligned} \tag{6}$$

$$\begin{aligned}\varphi_2 =& \frac{1}{3}\{\frac{\frac{2t}{\theta-1}(\frac{\theta}{\theta-1})^{1-2\theta}}{\frac{t}{\theta-1}(c_0+c_1)^{-\theta+1} + (\varepsilon-\varepsilon^2)c_0 t(c_0+c_1)^{1-\theta}(\frac{\theta}{\theta-1})^2 + \frac{t}{\theta-1}(c_1+(1-\varepsilon+\varepsilon^2)c_0)^{2-\theta}(\frac{\theta}{\theta-1})^2} \\ &+\frac{t*(c_1+c_0)^{1-\theta}}{\theta}(\frac{\theta}{\theta-1})^{1-\theta}(1-(\frac{\theta}{\theta-1})^{1-\theta}) + \frac{t(c_0+c_1)^{1-\theta}}{\theta}(\frac{\theta}{\theta-1})^{2-2\theta} + 2t*(\frac{\theta}{\theta-1})^{-\theta}(c_1+(1-\varepsilon+\varepsilon^2)c_0)^{1-\theta}(1+(\frac{\theta}{\theta-1})^{-\theta})\}\end{aligned} \tag{7}$$

$$\begin{aligned}\varphi_3 =& \frac{1}{3}(\frac{\theta}{\theta-1})^{-\theta}[\frac{2tc_0(\varepsilon-\varepsilon^2)(c_1+c_0)^{1-\theta}}{2\theta+1}(\theta+(\frac{\theta}{\theta-1})^{2-\theta}) + \frac{t}{\theta-t}\left(\frac{\theta}{\theta-1}\right)^{2-\theta}\left(\left(c_1+(1-\varepsilon-\varepsilon^2)c_0\right)^{2-\theta} - (c_1+c_0)^{2-\theta}\right) \\ &+(\varepsilon-\varepsilon^2)c_0 t(c_0+c_1)^{1-\theta}(\frac{\theta}{\theta-1})^{2-\theta} + \frac{2t}{\theta-1}((c_1+(1-\varepsilon+\varepsilon^2)c_0)^{1-\theta} - (c_1+c_0)^{1-\theta})]\end{aligned} \tag{8}$$

From the expressions of φ_1, φ_2 and φ_3, it can be seen that, the manufacturer's gain is heavily influenced by the condition of the reclaimed products for recycling. This result is reasonable because the condition of the reclaimed products influence manufacturing cost of new products, and the later will further affect the distributor's sales price and quantity, and the condition of reclaimed products for recycling affect the cost and quantity of reclaiming. In general, the condition of the reclaimed products for recycling has an impact on the closed-loop supply chain systemas a whole.

Next, this paper gives a numerical example to demonstrate the difference of the result between the modified Shapley value and the classical one.

4. Numerical Example

To verify the modified Shapley value has lower uncertainty in gains allocation of closed-loop supply chain under third-party reclaim mode, a numerical example is given below. Suppose the demand of the product market is: $q=400*p^{-2}$. The unit cost for distributor distributing products is 1. Assume if the manufacturer doesn't cooperate with the recycler, the manufacturer's unit cost of product is 4 and the reclaiming cost is $4\varepsilon^2$, and then if they co-operate by forming a coalition, the manufacturer's manufacturing cost is $4-4\varepsilon+4\varepsilon^2$.

Put the above numerical assumptions and $\varepsilon=0.5$ into the formula of classical Shapley value and get the gains allocation and variances of the manufacture, the distributor, and the recycler, respectively

$\varphi_1 = 9.1, \varphi_2 = 12.6, \varphi_3 = 3.3$; $R_1^2 = 20.6,\ R_2^2 = 18.2, R_3^2 = 20.$

Put the above numerical assumptions into the formula of modified Shapley value and get the gains allocation and variances of the manufacture, the distributor, and the recycler, respectively

$\varphi_1^{'} = 9.1, \varphi_2^{'} = 13.2, \varphi_3^{'} = 2.7$; $R_{01}^2 = 20.6, R_{02}^2 = 17.6, R_{03}^2 = 18.5$

First, from the allocation results, it can be seen that there is no change in manufacturer's gains, which is due to the coalition condition which has no influence on the manufacturer’s earning; and the modified Shapley value only cause the gains re-allocation between the distributor and recycler.

Second, from the uncertainty of the results, it can be seen that the result of the modified Shapley value has lower uncertainty (variance). Specifically, the uncertainty of manufacturers does not change, and the distributor and the recycler have got uncertainty dropped.

For different values of ε, through the above process the research gets the similar results and the same conclusions. It shows that although the condition of the reclaimed products has influence on all of the participants. But it does not change that the allocation results through the modified Shapley have lower uncertainty compared with the results through classical Shapley value.

5. Conclusion

As classical Shapley participants are assumed to join a particular coalition randomly and are subject to uniform distribution, which is inconsistent with the reality of the coalition forming process in the closed-loop supply chain under the third-party reclaim mode, so that a modified Shapley value is developed for gains allocation among the manufacture, the distributor and the recycler.

This research gets a general modified Shapley value allocation result of the closed-loop supply chain under third-party reclaim mode and then gives a numerical example. The numerical example shows that the results applying modified Shapley value suffer lower uncertainty.

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